Eur. Phys. J. C 46, 497–509 (2006)
Digital Object Identifier (DOI) 10.1140/epjc/s2006-02513-y

THE EUROPEAN PHYSICAL JOURNAL C

Phenomenology of the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ model with right-handed neutrinos

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Received: 12 October 2005 / Revised version: 21 November 2005 /
Published online: 24 March 2006 –

Abstract. A phenomenological analysis of the three-family model based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ with right-handed neutrinos is carried out. Instead of using the minimal scalar sector able to break the symmetry in a proper way, we introduce an alternative set of four Higgs scalar triplets, which combined with an anomaly-free discrete symmetry, produces a quark mass spectrum without hierarchies in the Yukawa coupling constants. We also embed the structure into a simple gauge group and show some conditions for achieving a low energy gauge coupling unification, avoiding possible conflict with proton decay bounds. By using experimental results from the CERN-LEP, SLAC linear collider, and atomic parity violation data, we update constraints on several parameters of the model.

PACS. 12.60.Cn; 12.15.Ff; 12.15.Mm

1 Introduction

Two intriguing puzzles completely unanswered in modern particle physics are the number of fermion families in nature, and the pattern of masses and mixing angles in the fermion sector. One interesting attempt to answer to the question of family replication is provided by the 3-3-1 extension of the local gauge symmetry $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ of the standard model (SM) of the strong and electroweak interactions [1]. This extension, based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, has among its best features that several models can be constructed so that anomaly cancellation is achieved by an interplay between the families, all of them under the condition $N_f = N_c = 3$, where N_f is the number of families and N_c is the number of colors of $SU(3)_c$ (three-family models) [2].

Two 3-3-1 three-family models have been extensively studied over the last decade [2, 3]. In one of them, the three known left-handed lepton components for each family are associated to three $SU(3)_L$ triplets as $(\nu_l, l^-, l^+)_L$, where l^+_L is related to the right-handed isospin singlet of the charged lepton l^-_L in the SM [2]. In the other model, the three $SU(3)_L$ lepton triplets are of the form $(\nu_l, l^-, \nu^c_l)_L$, where ν^c_l is related to the right-handed component of the neutrino field ν_l (a model with right-handed neutrinos) [3]. In the first model, anomaly cancellation implies quarks with the exotic electric charges $-4/3$ and $5/3$, while in the second one the extra particles have only ordinary electric charges.

Our aim in this paper is to do a phenomenological analysis of the 3-3-1 model in the version that includes right-handed neutrinos, including a detailed study of the fermion mass spectrum, with emphasis in the quark sector. Previous work [3] just presented the Yukawa Lagrangians without looking for constraints able to produce a consistent quark mass spectrum. It will be shown that a convenient set of four Higgs scalars, combined with an appropriate anomaly-free discrete Z_2 symmetry, produces an appealing quark mass spectrum without strong hierarchies for the Yukawa couplings. Furthermore, we shall study the embedding and unification of this gauge structure into $SU(6)$, which is an appropriate unification gauge group. Finally, we will set updated constraints on several parameters of the model.

The problem of lepton masses in the context of 3-3-1 three-family models has been studied, for example, in [4, 5], and we already know, from the analysis presented in [5–7], that models based on the 3-3-1 local gauge structure are suitable for describing some neutrino properties, because they include in a natural way most of the ingredients needed to explain the masses and mixing in the neutrino sector. In particular, [6] addresses this issue for the model studied here.

This paper is organized as follows. In Sect. 2 we review the model, introduce the new scalar sector, embed the structure into a covering group and calculate the charged and neutral electroweak currents. In Sect. 3 we study the charged fermion mass spectrum. In Sect. 4 we do the renormalization group equation analysis and show the conditions for the gauge coupling unification. In Sect. 5 we fix

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the new bounds on the mixing angle between the two flavor diagonal neutral currents present in the model, and discuss the constraints coming from violation of the unitarity of the Cabbibo–Kobayashi–Maskawa (CKM) quark-mixing matrix and from flavor changing neutral currents (FCNC). Finally, in the last section, we present our conclusions.

2 The model

The model that we are about to study here was sketched for the first time in the literature in the first reference in [3], with some phenomenology presented in the other four papers in the same reference. Some of the formulas quoted in the following sections are taken from those references and from [8]; corrections to some minor printing mistakes in the original papers are included.

2.1 The gauge group

As was stated above, the model that we are interested in is based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, which has 17 gauge bosons: one gauge field B^μ associated with $U(1)_X$, the eight gluon fields G^μ associated with $SU(3)_c$ which remain massless after breaking the symmetry, and another eight gauge fields associated with $SU(3)_L$ and for convenience written as [8]

$$\frac{1}{2}\lambda_\alpha A^\mu_\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} D^\mu_1 & W^{+\mu} & K^{+\mu} \\ W^{-\mu} & D^\mu_2 & K^{0\mu} \\ K^{-\mu} & \bar{K}^{0\mu} & D^\mu_3 \end{pmatrix} ,$$

where $D^\mu_1 = A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6}$, $D^\mu_2 = -A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6}$, and $D^\mu_3 = -2A^\mu_8/\sqrt{6}$. λ_i, $i = 1, 2, \ldots, 8$, are the eight Gell–Mann matrices normalized as $Tr(\lambda_i \lambda_j) = 2\delta_{ij}$.

The charge operator associated with the unbroken gauge symmetry $U(1)_Q$ is given by

$$Q = \frac{\lambda_{3L}}{2} + \frac{\lambda_{8L}}{2\sqrt{3}} + XI_3 , \tag{1}$$

where $I_3 = \mathrm{diag.}(1, 1, 1)$ is the diagonal 3×3 unit matrix, and the X values are related to the $U(1)_X$ hypercharge and are fixed by anomaly cancellation. The sine square of the electroweak mixing angle is given by

$$S^2_W = 3g_1^2/(3g_3^2 + 4g_1^2) , \tag{2}$$

where g_1 and g_3 are the gauge coupling constants of $U(1)_X$ and $SU(3)_L$, respectively, and the photon field is given by [3, 8]

$$A^\mu_0 = S_W A^\mu_3 + C_W \left[\frac{T_W}{\sqrt{3}} A^\mu_8 + \sqrt{(1 - T^2_W/3)} B^\mu\right] , \tag{3}$$

where C_W and T_W are the cosine and tangent of the electroweak mixing angle, respectively.

There are two weak neutral currents in the model associated with the two flavor diagonal neutral weak gauge bosons

$$Z^\mu_0 = C_W A^\mu_3 - S_W \left[\frac{T_W}{\sqrt{3}} A^\mu_8 + \sqrt{(1 - T^2_W/3)} B^\mu\right] ,$$
$$Z'^\mu_0 = -\sqrt{(1 - T^2_W/3)} A^\mu_8 + \frac{T_W}{\sqrt{3}} B^\mu , \tag{4}$$

and one current associated with the flavor nondiagonal electrically neutral gauge boson $K^{0\mu}$, which carries a kind of weak V-isospin charge. In the former expressions, Z^μ_0 coincides with the weak neutral current of the SM [3, 8]. Using (3) and (4), we realize that the gauge boson Y^μ associated with the abelian hypercharge in the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ SM gauge group is

$$Y^\mu = \frac{T_W}{\sqrt{3}} A^\mu_8 + \sqrt{(1 - T^2_W/3)} B^\mu . \tag{5}$$

2.2 The spin 1/2 particle content

The quark content for the three families in this model (known in the literature as the 3-3-1 model with right-handed neutrinos) is the following: $Q^i_L = (u^i, d^i, D^i)_L \sim (3, 3, 0)$, $i = 1, 2$ for two families, where D^i_L are two extra quarks of electric charge $-1/3$ (the numbers in parentheses stand for the $[SU(3)_c, SU(3)_L, U(1)_X]$ quantum numbers in that order); $Q^3_L = (d^3, u^3, U)_L \sim (3, 3^*, 1/3)$, where U_L is an extra quark of electric charge 2/3. The right-handed quarks are $u^{ac}_L \sim (3^*, 1, -2/3)$, $d^{ac}_L \sim (3^*, 1, 1/3)$ with $a = 1, 2, 3$ being a family index, $D^{ic}_L \sim (3^*, 1, 1/3)$, $i = 1, 2$, and $U^c_L \sim (3^*, 1, -2/3)$.

The lepton content is given by the three $SU(3)_L$ antitriplets $L_{lL} = (l^-, \nu^0_l, \nu^{0c}_l)_L \sim (1, 3^*, -1/3)$, for $l = e, \mu, \tau$ a leptonic family index, and the three singlets $l^+_L \sim (1, 1, 1)$, where ν^0_l is the neutrino field associated with the lepton l^-, and ν^{0c}_l plays the role of the right-handed neutrino field associated to the same flavor. Notice that this model does not contain exotic charged leptons, and universality for the known leptons in the three families is present at the tree-level in the weak basis.

With these quantum numbers it is just a matter of counting to check that the model is free of the following gauge anomalies [8]: $[SU(3)_c]^3$ (as in the SM, $SU(3)_c$ is vector-like); $[SU(3)_L]^3$ (six triplets and six antitriplets), $[SU(3)_c]^2U(1)_X$; $[SU(3)_L]^2U(1)_X$; $[\mathrm{grav}]^2U(1)_X$ and $[U(1)_X]^3$, where $[\mathrm{grav}]^2U(1)_X$ stands for the gravitational anomaly as described in [9].

2.3 The new scalar sector

Instead of using the set of three triplets of Higgs scalars introduced in the original papers [3], or the most economical set of two triplets introduced in [8] (none of them being able to produce a realistic mass spectrum), we propose here to work with the following set of four Higgs scalar fields, and vacuum expectation values (VEV):

$$\begin{aligned}
\langle\phi_1^T\rangle &= \langle(\phi_1^+,\phi_1^0,\phi_1^{'0})\rangle = \langle(0,0,v_1)\rangle \sim (1,3,1/3)\,,\\
\langle\phi_2^T\rangle &= \langle(\phi_2^+,\phi_2^0,\phi_2^{'0})\rangle = \langle(0,v_2,0)\rangle \sim (1,3,1/3)\,,\\
\langle\phi_3^T\rangle &= \langle(\phi_3^0,\phi_3^-,\phi_3^{'-})\rangle = \langle(v_3,0,0)\rangle \sim (1,3,-2/3)\,,\\
\langle\phi_4^T\rangle &= \langle(\phi_4^+,\phi_4^0,\phi_4^{'0})\rangle = \langle(0,0,V)\rangle \sim (1,3,1/3)\,,
\end{aligned} \quad (6)$$

with the hierarchy $v_1 \sim v_2 \sim v_3 \sim 10^2$ GeV $\ll V$. The analysis shows that this set of VEV breaks the $SU(3)_c\otimes SU(3)_L\otimes U(1)_X$ symmetry into two steps following the scheme

$$\begin{aligned}
&SU(3)_c\otimes SU(3)_L\otimes U(1)_X \stackrel{(V+v_1)}{\longrightarrow}\\
&SU(3)_c\otimes SU(2)_L\otimes U(1)_Y \stackrel{(v_2+v_3)}{\longrightarrow} SU(3)_c\otimes U(1)_Q\,,
\end{aligned}$$

which in turn allows for the matching conditions $g_2 = g_3$ and

$$\frac{1}{g_Y^2} = \frac{1}{g_1^2} + \frac{1}{3g_2^2}\,, \quad (7)$$

where g_2 and g_Y are the gauge coupling constants of the $SU(2)_L$ and $U(1)_Y$ gauge groups in the SM, respectively.

We will see in the next sections that this scalar structure properly breaks the symmetry, provides with masses for the gauge bosons and, combined with a discrete symmetry; it is enough to produce a consistent mass spectrum for the up and down quark sectors (a realistic mass spectrum in the lepton sector requires new ingredients as, for example, $SU(3)_L$ leptoquark scalar triplets and/or sextuplets, as we will briefly mention subsequently).

2.4 $SU(6)\supset SU(5)$ as a covering group

The Lie algebra of $SU(3)\otimes SU(3)\otimes U(1)$ is a maximal subalgebra of the simple algebra of $SU(6)$. The five fundamental irreducible representations (irreps) of $SU(6)$ are: $\{6\},\{6^*\},\{15\},\{15^*\}$ and $\{20\}$, which is real. The branching rules for these fundamental irreps into $SU(3)_c\otimes SU(3)_L\otimes U(1)_X$ are [10]:

$$\begin{aligned}
\{6\}&\rightarrow(3,1,-1/3)\oplus(1,3,1/3)\,,\\
\{15\}&\rightarrow(3^*,1,-2/3)\oplus(1,3^*,2/3)\oplus(3,3,0)\,,\\
\{20\}&\rightarrow(1,1,1)\oplus(1,1,-1)\oplus(3,3^*,1/3)\oplus(3^*,3,-1/3)\,,
\end{aligned}$$

where we have normalized the $U(1)_X$ hypercharge for our convenience.

From these branching rules and from the fermion structure presented above, it is clear that all the particles in the 3-3-1 model with right-handed neutrinos can be included in the following $SU(6)$ reducible representation

$$5\{6^*\}+3\{20\}+4\{15\}\,, \quad (8)$$

which includes such new exotic particles as, for example,

$$\begin{aligned}
(N^0,E^+,E'^+)_L &\sim (1,3^*,2/3)\subset\{15\}\,,\\
E_L^- &\sim (1,1,-1)\subset\{20\}\,,\\
(D'^c,U'^c,U''^c)_L &\sim (3^*,3,-1/3)\subset\{20\}\,.
\end{aligned}$$

The analysis reveals that the reducible representation in (8) is anomalous. The simplest $SU(6)$ reducible representation, which is free of anomalies and includes the fields in (8), is given by [10]

$$8\{6^*\}+3\{20\}+4\{15\}\,, \quad (9)$$

which also includes the following new exotic particles (all with ordinary electric charges): four families of 3-3-1 up-type and down-type quarks, four more exotic down-type quarks, plus eight families of 3-3-1 lepton triplets, among a good deal of other particles.

It is clear from the following decomposition of irrep $\{6^*\}$ of $SU(6)$ into $SU(5)\otimes U(1)$

$$\begin{aligned}
\{6^*\} &= \{d^c,-N_E^0,E^-,N_E^{0c}\}_L\\
&\longrightarrow \{d^c,-N_E^0,E^-\}_L\oplus N_{EL}^{0c}\,,
\end{aligned} \quad (10)$$

that for $N_{EL}^0=\nu_{eL}$ and $E_L^-=e_L^-$, we obtain the known $SU(5)$ model of Georgi and Glashow [11]; so, in some sense, this model is an extension of one of the first grand unified theories (GUT) presented in the literature.

2.5 The gauge boson sector

After breaking the symmetry with $\langle\phi_i\rangle$, $i=1,\ldots,4$, and using the covariant derivative for triplets $D^\mu=\partial^\mu - ig_3\lambda_{\alpha L}A_\alpha^\mu/2 - ig_1XB_\mu I_3$, we obtain the following mass terms in the gauge boson sector.

2.5.1 Spectrum in the charged gauge boson sector

A straightforward calculation shows that the charged gauge bosons $K_\mu^\pm$ and $W_\mu^\pm$ do not mix with each other and have the following masses: $M_{K^\pm}^2 = g_3^2(V^2+v_1^2+v_3^2)/2$ and $M_W^2 = g_3^2(v_2^2+v_3^2)/2$, which for $g_3=g_2$ and using the experimental value $M_W = 80.423\pm0.039$ GeV [12] implies $\sqrt{v_2^2+v_3^2}\simeq 175$ GeV. In the same way, $K^{0\mu}$ (and its antiparticle $\bar{K}^{0\mu}$) does not mix with the other two electrically neutral gauge bosons and gets a bare mass $M_{K^0}^2 = g_3^2(V^2+v_1^2+v_2^2)/2 \approx M_{K^\pm}^2$. Notice that v_1 does not contribute to the $W^\pm$ mass because it is associated with an $SU(2)_L$ singlet Higgs scalar.

2.5.2 Spectrum in the neutral gauge boson sector

The algebra now shows that in this sector, the photon field A_0^μ in (3) decouples from Z_0^μ and $Z_0'^\mu$ and remains massless. Then, in the basis $(Z_0^\mu, Z_0'^\mu)$, we obtain the following 2×2 mass matrix

$$\frac{\eta^2 g_3^2}{4C_W^2}\begin{pmatrix} \frac{v_2^2+v_3^2}{\eta^2} & \frac{v_2^2C_{2W}-v_3^2}{\eta} \\ \frac{v_2^2C_{2W}-v_3^2}{\eta} & v_2^2C_{2W}^2+v_3^2+4(V^2+v_1^2)C_W^4 \end{pmatrix}\,, \quad (11)$$

where $C_{2W} = C_W^2 - S_W^2$ and $\eta^{-2} = (3 - 4S_W^2)$. This matrix provides a mixing between Z_0^μ and $Z_0'^\mu$ of the form

$$\tan(2\theta) = \frac{2\sqrt{(3-4S_W^2)}(v_2^2 C_{2W} - v_3^2)}{4C_W^4(V^2+v_1^2) - 2v_3^2 C_{2W} - v_2^2(3-4S_W^2 - C_{2W}^2)} \xrightarrow{V\to\infty} 0 . \quad (12)$$

The physical fields are then

$$Z_1^\mu = Z_0^\mu \cos\theta - Z_0'^\mu \sin\theta ,$$
$$Z_2^\mu = Z_0^\mu \sin\theta + Z\prime_0^\mu \cos\theta .$$

An updated bound on the mixing angle θ will be calculated in Sect. 5 using experimental results.

2.6 Currents

2.6.1 Charged currents

The Hamiltonian for the currents, charged under the generators of the $SU(3)_L$ group, is $H^{\mathrm{CC}} = g_3(W_\mu^+ J_{W^+}^\mu + K_\mu^+ J_{K^+}^\mu + K_\mu^0 J_{K^0}^\mu)/\sqrt{2} + h.c.$, with

$$J_{W^+}^\mu = \left(\sum_{i=1}^{2} \bar{u}_L^i \gamma^\mu d_L^i\right) - \bar{u}_L^3 \gamma^\mu d_L^3 - \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma^\mu l_L^- ,$$
$$J_{K^+}^\mu = \left(\sum_{i=1}^{2} \bar{u}_L^i \gamma^\mu D_L^i\right) - \bar{U}_L \gamma^\mu d_L^3 - \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}^{0c} \gamma^\mu l_L^- ,$$
$$J_{K^0}^\mu = \left(\sum_{i=1}^{2} \bar{d}_L^i \gamma^\mu D_L^i\right) - \bar{U}_L \gamma^\mu u_L^3 - \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}^{0c} \gamma^\mu \nu_{lL} ,$$

where K_μ^0 is an electrically neutral gauge boson but carries a kind of weak V-isospin charge, besides its flavor nondiagonal.

2.6.2 Neutral currents

The neutral currents $J_\mu(EM)$, $J_\mu(Z)$ and $J_\mu(Z')$, associated with the Hamiltonian

$$H^0 = eA^\mu J_\mu(EM) + (g_3/C_W) Z^\mu J_\mu(Z) + (g_1/\sqrt{3}) Z'^\mu J_\mu(Z') \quad (13)$$

are [3]

$$J_\mu(EM) = \frac{2}{3}\left[\sum_{a=1}^{3} \bar{u}_a \gamma_\mu u_a + \bar{U}\gamma_\mu U\right] - \frac{1}{3}\left[\sum_{a=1}^{3} \bar{d}^a \gamma_\mu d^a + \sum_{i=1}^{2} \bar{D}^i \gamma_\mu D^i\right] - \sum_{l=e,\mu,\tau} \bar{l}^- \gamma_\mu l^- = \sum_f q_f \bar{f} \gamma_\mu f ,$$

$$J_\mu(Z) = J_{\mu,L}(Z) - S_W^2 J_\mu(EM) ,$$
$$J_\mu(Z') = -J_{\mu,L}(Z') + T_W J_\mu(EM) ,$$

where $e = g_3 S_W = g_1 C_W \sqrt{(1 - T_W^2/3)} > 0$ is the electric charge, q_f is the electric charge of the fermion f in units of e, and $J_\mu(EM)$ is the electromagnetic current.

The left-handed currents are

$$J_{\mu,L}(Z) = \frac{1}{2}\Big[\sum_{a=1}^{3}(\bar{u}_L^a \gamma_\mu u_L^a - \bar{d}_L^a \gamma_\mu d_L^a) + \sum_{l=e,\mu,\tau} (\bar{\nu}_{lL}\gamma_\mu \nu_{lL} - \bar{l}_L^- \gamma_\mu l_L^-)\Big] = \sum_F \bar{F}_L T_{3f} \gamma_\mu F_L , \quad (14)$$

$$\begin{aligned} J_{\mu,L}(Z') = {} & S_{2W}^{-1}\Big[\bar{u}_{1L}\gamma_\mu u_{1L} + \bar{u}_{2L}\gamma_\mu u_{2L} - \bar{d}_{3L}\gamma_\mu d_{3L} - \sum_l (\bar{l}_L^- \gamma_\mu l_L^-)\Big] \\ & + T_{2W}^{-1}\Big[\bar{d}_{1L}\gamma_\mu d_{1L} + \bar{d}_{2L}\gamma_\mu d_{2L} - \bar{u}_{3L}\gamma_\mu u_{3L} - \sum_l (\bar{\nu}_{lL}\gamma_\mu \nu_{lL})\Big] \\ & + T_W^{-1}\Big[\bar{D}_{1L}\gamma_\mu D_{1L} + \bar{D}_{2L}\gamma_\mu D_{2L} - \bar{U}_L \gamma_\mu U_L - \sum_l (\bar{\nu}_{lL}^{0c}\gamma_\mu \nu_{lL}^{0c})\Big] \\ = {} & \sum_F \bar{F}_L T'_{3f} \gamma_\mu F_L , \end{aligned} \quad (15)$$

where $S_{2W} = 2S_W C_W$, $T_{2W} = S_{2W}/C_{2W}$, $T_{3f} = Dg(1/2, -1/2, 0)$ is the third component of the weak isospin, $T'_{3f} = Dg(S_{2W}^{-1}, T_{2W}^{-1}, -T_W^{-1})$ is a convenient 3×3 diagonal matrix, both acting on the representation 3 of $SU(3)_L$ (the negative value when acting on the representation 3^*, which is also true for the matrix T_{3f}) and F is a generic symbol for the representations 3 and 3^* of $SU(3)_L$. Notice that $J_\mu(Z)$ is the neutral current of the SM (with the extra fields included in $J_\mu(EM)$). This allows us to identify Z_μ as the neutral gauge boson of the SM, which is consistent with (4) and (5).

The couplings of the flavor diagonal mass eigenstates Z_1^μ and Z_2^μ are given by

$$H^{\mathrm{NC}} = \frac{g_3}{2C_W}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\bar{f}\gamma_\mu[a_{iL}(f)(1-\gamma_5) + a_{iR}(f)(1+\gamma_5)]f\} = \frac{g_3}{2C_W}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\bar{f}\gamma_\mu[g(f)_{iV} - g(f)_{iA}\gamma_5]f\} ,$$

with

$$\begin{aligned} a_{1L}(f) &= \cos\theta(T_{3f} - q_f S_W^2) + \Theta\sin\theta(T'_{3f} - q_f T_W), \\ a_{1R}(f) &= -q_f\left(\cos\theta S_W^2 + \Theta\sin\theta T_W\right), \\ a_{2L}(f) &= \sin\theta(T_{3f} - q_f S_W^2) - \Theta\cos\theta(T'_{3f} - q_f T_W) , \\ a_{2R}(f) &= -q_f\left(\sin\theta S_W^2 - \Theta\cos\theta T_W\right) , \end{aligned} \quad (16)$$

where $\Theta = S_W C_W / \sqrt{(3 - 4S_W^2)}$. From these coefficients we can read

$$\begin{aligned} g(f)_{1V} &= \cos\theta (T_{3f} - 2q_f S_W^2) \\ &\quad + \Theta \sin\theta (T'_{3f} - 2q_f T_W)\ , \\ g(f)_{2V} &= \sin\theta (T_{3f} - 2q_f S_W^2) \\ &\quad - \Theta \cos\theta (T'_{3f} - 2q_f T_W)\ , \\ g(f)_{1A} &= \cos\theta T_{3f} + \Theta \sin\theta T'_{3f}\ , \\ g(f)_{2A} &= \sin\theta T_{3f} - \Theta \cos\theta T'_{3f}\ . \end{aligned} \tag{17}$$

The values of g_{iV} and g_{iA}, with $i = 1,2$, are listed in Tables 1 and 2 [3].

As we can see, in the limit $\theta = 0$ the couplings of Z_1^μ to the ordinary leptons and quarks are the same as in the SM; due to this property, we can test the new physics beyond the SM predicted by this particular model.

3 Fermion masses

The Higgs scalars introduced in Sect. 2 break the symmetry in an appropriate way and, at the same time, produce mass terms for the fermion fields via Yukawa interactions.

In order to restrict the number of Yukawa couplings, and produce a realistic mass spectrum, we introduce an anomaly-free discrete Z_2 symmetry [13] with the following assignments of charges:

$$\begin{aligned} Z_2(Q_L^a, \phi_2, \phi_3, \phi_4, u_L^{ic}, d_L^{ac}) &= 1 \\ Z_2(\phi_1, u_L^{3c}, U_L^c, D_L^{ic}, L_{lL}, l_L^+) &= 0\ , \end{aligned} \tag{18}$$

where $a = 1,2,3,\ i = 1,2$ and $l = e, \mu, \tau$ are family indexes as above.

Table 1. The $Z_1^\mu \longrightarrow \bar{f} f$ couplings

f	$g(f)_{1V}$	$g(f)_{1A}$
$u^{1,2}$	$(\frac{1}{2} - 4S_W^{\frac{2}{3}}) \cos\theta + \Theta(s_{2W}^{-1} - \frac{4T_W}{3}) \sin\theta$	$\frac{1}{2}\cos\theta + \Theta S_{2W}^{-1} \sin\theta$
u^3	$(\frac{1}{2} - 4S_W^{\frac{2}{3}}) \cos\theta - \Theta(T_{2W}^{-1} + \frac{4T_W}{3}) \sin\theta$	$\frac{1}{2}\cos\theta - \Theta T_{2W}^{-1} \sin\theta$
$d^{1,2}$	$(-\frac{1}{2} + \frac{2S_W^2}{3}) \cos\theta + \Theta(T_{2W}^{-1} + \frac{2T_W}{3}) \sin\theta$	$-\frac{1}{2}\cos\theta + \Theta T_{2W}^{-1} \sin\theta$
d^3	$(-\frac{1}{2} + \frac{2S_W^2}{3}) \cos\theta - \Theta(S_{2W}^{-1} - \frac{2T_W}{3}) \sin\theta$	$-\frac{1}{2}\cos\theta - \Theta S_{2W}^{-1} \sin\theta$
U	$-\frac{4S_W^2}{3} \cos\theta - \Theta(T_W^{-1} + \frac{4T_W}{3}) \sin\theta$	$\Theta T_W^{-1} \sin\theta$
$D^{1,2}$	$\frac{2S_W^2}{3} \cos\theta + \Theta(T_W^{-1} + \frac{2S_W^2}{3}) \sin\theta$	$-\Theta T_W^{-1} \sin\theta$
e, μ, τ	$(-\frac{1}{2} + 2S_W^2) \cos\theta - \Theta(S_{2W}^{-1} - 2T_W) \sin\theta$	$-\frac{1}{2}\cos\theta - \Theta S_{2W}^{-1} \sin\theta$
ν_e, ν_μ, ν_τ	$\frac{1}{2}\cos\theta - \Theta T_{2W}^{-1} \sin\theta$	$\frac{1}{2}\cos\theta - \Theta T_{2W}^{-1} \sin\theta$
$\nu_e^{0c}, \nu_\mu^{0c}, \nu_\tau^{0c}$	$-\Theta T_W^{-1} \sin\theta$	$-\Theta T_W^{-1} \sin\theta$

Table 2. The $Z_2^\mu \longrightarrow \bar{f} f$ couplings

f	$g(f)_{2V}$	$g(f)_{2A}$
$u^{1,2}$	$(\frac{1}{2} - \frac{4S_W^2}{3}) \sin\theta - \Theta(S_{2W}^{-1} - \frac{4T_W}{3}) \cos\theta$	$\frac{1}{2}\sin\theta - \Theta S_{2W}^{-1} \cos\theta$
u^3	$(\frac{1}{2} - \frac{4S_W^2}{3}) \sin\theta + \Theta(T_{2W}^{-1} + \frac{4T_W}{3}) \cos\theta$	$\frac{1}{2}\sin\theta + \Theta T_{2W}^{-1} \cos\theta$
$d^{1,2}$	$(-\frac{1}{2} + \frac{2S_W^2}{3}) \sin\theta - \Theta(T_{2W}^{-1} + \frac{2T_W}{3}) \cos\theta$	$-\frac{1}{2}\sin\theta - \Theta T_{2W}^{-1} \cos\theta$
d^3	$(-\frac{1}{2} + \frac{2S_W^2}{3}) \sin\theta + \Theta(S_{2W}^{-1} - \frac{2T_W}{3}) \cos\theta$	$-\frac{1}{2}\sin\theta + \Theta S_{2W}^{-1} \cos\theta$
U	$-\frac{4S_W^2}{3} \sin\theta + \Theta(T_W^{-1} + \frac{4T_W}{3}) \cos\theta$	$\Theta T_W^{-1} \cos\theta$
$D^{1,2}$	$\frac{2S_W^2}{3} \sin\theta - \Theta(T_W^{-1} + \frac{2T_W}{3}) \cos\theta$	$-\Theta T_W^{-1} \cos\theta$
e, μ, τ	$(-\frac{1}{2} + 2S_W^2) \sin\theta + \Theta(S_{2W}^{-1} - \frac{2T_W}{3}) \cos\theta$	$-\frac{1}{2}\sin\theta + \Theta S_{2W}^{-1} \cos\theta$
ν_e, ν_μ, ν_τ	$\frac{1}{2}\sin\theta + \Theta T_{2W}^{-1} \cos\theta$	$\frac{1}{2}\sin\theta + \Theta T_{2W}^{-1} \cos\theta$
$\nu_e^{0c}, \nu_\mu^{0c}, \nu_\tau^{0c}$	$\Theta T_W^{-1} \cos\theta$	$\Theta T_W^{-1} \cos\theta$

3.1 The up quark sector

The most general invariant Yukawa Lagrangian for the up quark sector is given by

$$\mathcal{L}_Y^u = \sum_{\alpha=1,2,4} Q_L^3 \phi_\alpha C(h_\alpha^U U_L^c + \sum_{a=1}^{3} h_{a\alpha}^u u_L^{ac}) + \sum_{i=1}^{2} Q_L^i \phi_3^* C(\sum_{a=1}^{3} h_{ia}^u u_L^{ac} + h_i^{\prime U} U_L^c) + h.c. \ , \tag{19}$$

where the h's are Yukawa coupling constants and C is the charge conjugation operator.

Then, in the basis (u^1, u^2, u^3, U), and using the Z_2 symmetry, we get from (18), (19) the following tree-level up quark mass matrix

$$M_u = \begin{pmatrix} 0 & 0 & 0 & h_{11}^u v_1 \\ 0 & 0 & 0 & h_{21}^u v_1 \\ h_{13}^u v_3 & h_{23}^u v_3 & h_{32}^u v_2 & h_{34}^u V \\ h_1^{\prime U} v_3 & h_2^{\prime U} v_3 & h_2^U v_2 & h_4^U V \end{pmatrix}, \tag{20}$$

which is a rank one see-saw type mass matrix. As a matter of fact, analytical and numerical analysis of this matrix shows that $M_u^\dagger M_u$ has one eigenvalue equal to zero related to the eigenvector $[(h_{32}^u h_2^{\prime U} - h_{23}^u h_2^U), (h_{13}^u h_2^U - h_{32}^u h_1^{\prime U}), (h_{23}^u h_1^{\prime U} - h_{13}^u h_2^{\prime U}), 0]$, which we may identify with the up quark u in the first family, which remains massless at the tree-level.

In what follows, and without loss of generality, we shall impose the condition $v_1 = v_2 = v_3 \equiv v \ll V$, with the value for v fixed by the mass of the charged weak gauge boson $M_{W^\pm}^2 = g_3^2(v_2^2 + v_3^2)/2 = g_3^2 v^2$, which implies $v \approx 175/\sqrt{2} = 123$ GeV. Also, in order to simplify the otherwise cumbersome calculations and to avoid proliferation of unnecessary parameters at this stage of the analysis, we propose to start with the following simple matrix

$$M_u' = hv \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & h_{32}^u/h\delta^{-1} \\ 1 & 1 & 1 & \delta^{-1} \end{pmatrix}, \tag{21}$$

where $\delta = v/V$ is a perturbation expansion parameter and h is a parameter that can take any value of order one.

Neglecting terms of order δ^5 and higher, the four eigenvalues of $M_u'^\dagger M_u'$ are: one zero eigenvalue related to the eigenstate $(u^1 - u^2)/\sqrt{2}$ (notice the maximal mixing present); one see-saw eigenvalue $4h^2V^2\delta^4 = 4h^2v^2\delta^2 \approx m_c^2$ associated to the charm quark mass, and two tree-level values that we identify with the masses of the top quark t and the heavy quark U given, respectively, by

$$\frac{h^2V^2\delta^2}{2}[e_-^2 + \delta^2 e_+^2(4 - e_-^2)/4] \approx \frac{v^2}{2}(h - h_{32}^u)^2 \approx m_t^2 \ ;$$
$$h^2V^2[2 + \delta^2(6 + e_+/2) + \delta^4(4e_+^2 - e_+^2 e_-^2 - 32)/8] \approx m_U^2 \ ,$$

where $e_\pm = (1 \pm h_{32}^u/h)$.

So, in the up quark sector, the heavy quark gets a large mass of order V (the 3-3-1 scale), the top quark gets a mass at the electroweak scale [times a difference of Yukawa couplings that in the general case of the matrix (20) is $(h_2^U - h_{32}^u)$], the charm quark gets a see-saw mass, and the first family up quark u remains massless at the tree-level. From the former expressions, and using $m_t \approx 175$ GeV [12], we get $|h_2^U - h_{32}^u| \sim 2$ and $m_c \approx 2hv^2/V$, which implies $V \approx hm_t^2/m_c \approx 19.4h$TeV, fixing in this way an upper limit for the 3-3-1 mass scale.

The consistency of this model requires finding a mechanism that is able to produce a mass for the up quark u in the first family. A detailed study of the Lagrangian in (19) and the discrete symmetry used, allows us to draw the radiative diagram in Fig. 1, which is the only diagram available to produce one-loop radiative corrections in the quark subspace (u^1, u^2). The mixing in the Higgs sector in the diagram comes from a term in the scalar potential of the form $\lambda_{13}(\phi_1^*\phi_1)(\phi_3^*\phi_3)$, which turns on the radiative corrections.

In the analysis we must be careful because, in order to have a contribution different from zero, we must avoid maximal mixing in the first two weak interaction states, otherwise a submatrix of the democratic type arises. This is simply done by taking $h_{11}^u = 1 - k$ and $h_1^{\prime U} = 1 + k$ in the matrix (20), where k must be a small parameter in order to guarantee the see-saw character of the matrix for the up quarks.

When we evaluate the contribution coming from the diagram in Fig. 1, we get a finite value given by

$$\Delta_{ji} = N_{ji}[M^2 m_1^2 \ln(M^2/m_1^2) - M^2 m_3^2 \ln(M^2/m_3^2) + m_3^2 m_1^2 \ln(m_1^2/m_3^2)] \ , \tag{22}$$

where

$$N_{ji} = h_j^{\prime U} h_{i1}^u \lambda_{13} \times \frac{v_1 v_3 M}{16\pi^2(m_3^2 - m_1^2)(M^2 - m_1^2)(M^2 - m_3^2)} \ . \tag{23}$$

$M = h_4^U V$ is the mass vertex of the heavy exotic up quark, and m_1 and m_3 are the masses of $\phi_1^{\prime 0}$ and ϕ_3^0, respectively. To estimate the contribution given by this diagram, we assume the validity of the "extended survival hypothesis" [14] which, in our case, means $m_1 \approx m_3 \approx vllV \approx M$, which in turn implies a mass value

$$m_u \approx \lambda_{13} v\delta \ln(V/v)/8\pi^2 \approx 0.85\lambda_{13}\text{MeV} \ ,$$

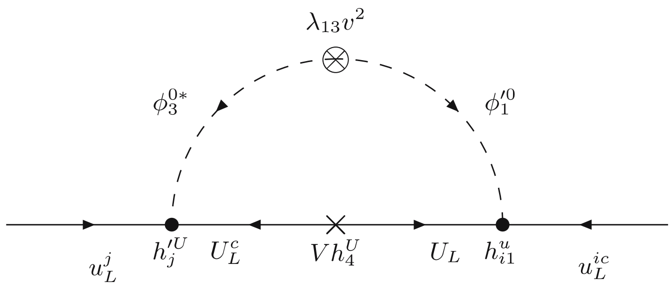

Fig. 1. One-loop diagram contributing to the radiative generation of the up quark mass

that for $\lambda_{13} \sim 2$ produces $m_u \approx 1.7$ MeV, which is of the correct order of magnitude [12] (a result that is independent of the value of k in the first approximation).

Notice that for $k \neq 0$, the state related to the u quark loses its maximal mixing, now becoming $\{-(h-h_{32})u^1 + [h-h_{32}(1-k)]u^2 + ku^3\}/N$, with N being a normalization factor. A value for k can be estimated using the Cabbibo angle.

3.2 The down quark sector

The most general Yukawa terms for the down quark sector, using the four Higgs scalars introduced in (6), are

$$\mathcal{L}_Y^d = \sum_{\alpha=1,2,4} \sum_i Q_L^i \phi_\alpha^* C \left(\sum_a h_{ia\alpha}^d d_L^{ac} + \sum_j h_{ij\alpha}^D D_L^{jc} \right) + Q_L^3 \phi_3 C \left(\sum_i h_i^D D_L^{ic} + \sum_a h_a^d d_L^{ac} \right) + h.c.. \quad (24)$$

In the basis $(d^1, d^2, d^3, D^1, D^2)$ and using the discrete symmetry Z_2, this expression produces the following tree-level down quark mass matrix

$$M_d = \begin{pmatrix} 0 & 0 & 0 & h_{11}^d v_1 & h_{21}^d v_1 \\ 0 & 0 & 0 & h_{12}^d v_1 & h_{22}^d v_1 \\ 0 & 0 & 0 & h_{13}^d v_1 & h_{23}^d v_1 \\ h_{11}^D v_2 & h_{21}^D v_2 & h_1^D v_3 & h_{114}^D V & h_{214}^D V \\ h_{12}^D v_2 & h_{22}^D v_2 & h_2^D v_3 & h_{124}^D V & h_{224}^D V \end{pmatrix}, \quad (25)$$

where we have used $h_{ia\alpha}^{D(d)} v_\alpha = h_{ia}^{D(d)} v_\alpha$.

The matrix M_d is again a see-saw type mass matrix, with at least one eigenvalue equal to zero, which gives many physical possibilities, depending upon the particular values assigned to the Yukawa couplings. For example, if all the Yukawa couplings are different from each other, then the matrix $M_d^\dagger M_d$ has rank one with one zero eigenvalue related to the eigenvector $[(h_{22}^D h_1^D - h_2^D h_{21}^D), (h_{11}^D h_{124}^D - h_{12}^D h_{114}^D), (h_{21}^D h_{12}^D - h_{11}^D h_{22}^D), 0, 0]$, which we may identify with the down quark d in the first family (which in any case remains massless at the tree-level). For this case, the general analysis shows that we have two see-saw eigenvalues associated with the bottom b and strange s quarks, the first one being enhanced by the sum of Yukawa couplings and the second one being suppressed by differences.

In the particular case when all the Yukawa couplings are equal to one but $h_{114}^D = h_{224}^D \equiv H^D \neq 1$, the null space of $M_d^\dagger M_d$ has rank two, with the eigenvectors associated with the zero eigenvalues given by $[-2,1,1,0,0]/\sqrt{6}$ and $[0,-1,1,0,0]/\sqrt{2}$, which implies only one see-saw eigenvalue associated with the bottom quark b with a mass value $m_b \approx 6v\delta/(1+H^D) \approx 3m_c/[h(1+H^D)]$, and with masses for the two heavy states of the order of $V(1 \pm H^D)$.

For the first case analyzed in the previous paragraph, the chiral symmetry remaining at tree-level is $SU(2)_f$ (quarks u and d are massless), and for the second case the chiral symmetry is $SU(3)_f$ (quarks u, d and s are massless). In both cases, the chiral symmetry will be broken by the radiative corrections.

In any case, a realistic analysis of the down sector requires bearing in mind the mixing matrix of the up quark sector and the fact that the CKM mixing matrix is almost diagonal and unitary. Aiming at this and in order to avoid a proliferation of parameters again, let us analyze the particular case given by the following left-right symmetric (Hermitian) down quark mass matrix

$$M_d' = h'v \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & f & g \\ 1 & 1 & f & H^D\delta^{-1} & \delta^{-1} \\ 1 & 1 & g & \delta^{-1} & H^D\delta^{-1} \end{pmatrix}, \quad (26)$$

where f and g are parameters of order one. This is the most general Hermitian mass matrix with only one zero eigenvalue related with the state $(d^1 - d^2)/\sqrt{2}$ (again maximal mixing, as required in order to end up with an almost diagonal and unitary CKM mixing matrix).

The two see-saw exact eigenvalues of M_d' are

$$-h'v\frac{\delta}{4}\left\{ \left[\frac{(f-g)^2}{H^D-1} + \frac{8+(f+g)^2}{1+H^D}\right] \pm \sqrt{\left[\frac{(f-g)^2}{H^D-1} + \frac{8+(f+g)^2}{1+H^D}\right]^2 - \frac{8(f-g)^4}{1-(H^D)^2}} \right\}. \quad (27)$$

Moreover, notice that for the particular case $g = -f$ (which implies some Yukawa couplings becoming complex), the five eigenvalues of the Hermitian matrix above yield the following simple exact analytical expressions

$$\frac{h'\delta^{-1}v}{2}\left[0, H_+^D(1 \pm \sqrt{1+16\delta^2/(H_+^D)^2}), H_-^D(1 \pm \sqrt{1+8f^2\delta^2/(H_-^D)^2})\right], \quad (28)$$

where $H_\pm^D = 1 \pm H^D$. The two see-saw values are thus $4\delta/H_+^D$ and $2\delta f^2/H_-^D$; which imply $f^2 h'/h \approx m_b H_-^D/m_c$ and $2h'/h \approx H_+^D m_s/m_c$, which can be seen as either a mild hierarchy between h and h', or implying a detailed tuning of some of the parameters of the order of one. The mass of the two heavy states is proportional to $h'VH_\pm^D$.

Again, radiative diagrams producing a nonzero mass for the down quark d in the first family must be found. For this purpose, we have the four diagrams depicted in Fig. 2 (two for D^1 and other two for D^2 in the heavy quark propagator). The mixing in the Higgs sector comes from terms in the scalar potential of the form $(f_1\phi_1\phi_3\phi_4 + f_2\phi_1\phi_2\phi_3 + h.c.)$. Now the algebra shows that

$$m_d \approx 2(f_1+f_2)\delta\ln(V/v)/8\pi^2, \quad (29)$$

which for $f_1 = f_2 \approx v$ implies $m_d \approx 2m_u$ without introducing a new mass scale in the model.

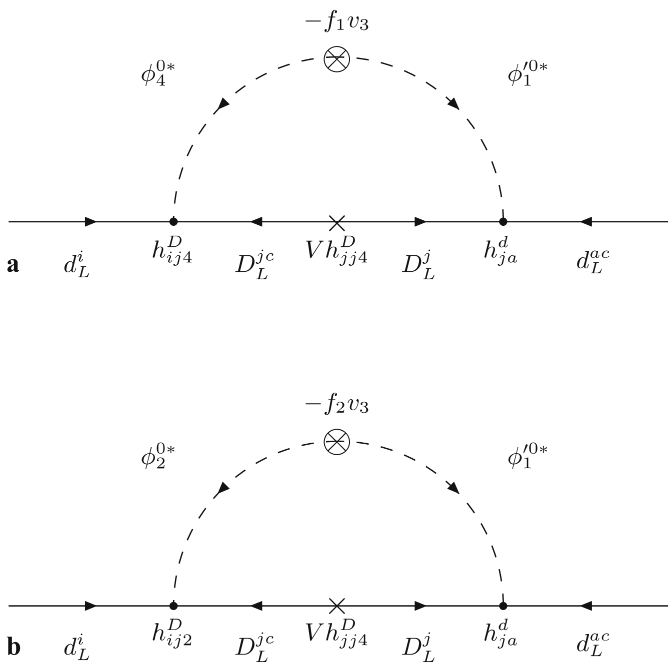

Fig. 2. One loop diagrams contributing to the radiative generation of the down quark mass

3.3 The lepton sector

Following the spirit of the analysis in the quark sector, and in order to avoid hierarchies in the Yukawa couplings, we introduce the discrete Z_2 symmetry in (18) in order to avoid terms proportional to

$$h^e_{ab} L_{aL} \phi_3 C e_{bL} + h'^e_{ab} \epsilon_{\alpha\beta\gamma} L^\alpha_{aL} L^\beta_{bL} \phi_3^\gamma + h.c.$$

in the Yukawa Lagrangian. Then, in order to generate masses for the charged leptons we must include either leptoquark Higgs field triplets if we want to use the radiative mechanism, or exotic leptons if we want to use the see-saw mechanism. For example, in [5] a singlet exotic charged lepton is introduced in the Pleitez–Frampton model [2] in order to implement the see-saw mechanism in the lepton sector. This analysis however is outside the scope of the study presented here.

In a similar way, masses for the neutrinos can be generated by introducing either new scalar fields, or new neutral exotic Weyl fermions. For example, in the context of the model studied here, a Majorana mass for the neutrinos can be generated by using scalars belonging to irrep $\{6\}$ of $SU(3)_L$. These scalars can be written as a 3×3 symmetric tensor

$$\chi_{\alpha\beta} = \begin{pmatrix} \chi_{11}^{4/3+X} & \chi_{12}^{1/3+X} & \chi_{13}^{1/3+X} \\ & \chi_{22}^{-2/3+X} & \chi_{23}^{-2/3+X} \\ & & \chi_{33}^{-2/3+X} \end{pmatrix} \sim (1, 6, X) \;, \tag{30}$$

where the upper symbol stands for the electric charge. Clearly, a VEV of the form $\langle \chi_{33}^0(1,6,2/3) \rangle \sim M$ produces a Majorana mass term of the form $M \nu^{0c}_{lL} \nu^{0c}_{l'L}$, a VEV of the form $\langle \chi_{22}^0(1,6,2/3) \rangle \sim w$ produces a Majorana mass term of the form $w \nu^0_{lL} \nu^0_{l'L}$, and a VEV of the form $\langle \chi_{23}^0(1,6,2/3) \rangle \sim m$ produces a Dirac mass term for the neutrinos. This issue is studied, for example, in [6, 7], where $SU(3)_L$ scalar singlets, triplets and sextuplets are used in order to provide the model with a realistic neutrino mass spectrum.

4 Gauge coupling unification

In a field theory, the coupling constants are defined as effective values which are energy scale dependent according to the renormalization group equation. In the modified minimal subtraction scheme [15], which we adopt in the following, the one loop renormalization group equation (RGE) for $\alpha = g^2/4\pi$ reads

$$\mu \frac{d\,\alpha}{d\,\mu} \simeq -b\alpha^2 \;, \tag{31}$$

where μ is the energy at which the coupling constant α is evaluated. The constant value b, called the beta function, is completely determined by the particle content of the model by

$$2\pi b = \frac{11}{6} C\,(\text{vectors}) - \frac{2}{6} C\,(\text{fermions}) - \frac{1}{6} C\,(\text{scalars}) \;,$$

where $C(\dots)$ is the group theoretical index of the representation inside the parentheses (we are assuming Weyl fermions and complex scalar fields [10]).

For the energy interval $m_Z < \mu < M_G$, the one loop solutions to the RGE (31) for the three SM gauge coupling constants are

$$\alpha_i^{-1}(m_Z) = \frac{\alpha_i^{-1}(M_G)}{c_i} - b_i(F,H) \ln\left(\frac{M_G}{m_Z}\right) , \tag{32}$$

where $i = Y, 2, c$ refers to the coupling constants of $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$, respectively, with the beta functions given by

$$2\pi \begin{pmatrix} b_Y \\ b_2 \\ b_c \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{22}{3} \\ 11 \end{pmatrix} - \begin{pmatrix} \frac{20}{9} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} F - \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 0 \end{pmatrix} H, \tag{33}$$

where F is the number of families contributing to the beta functions and H is the number of low energy $SU(2)_L$ scalar field doublets ($H = 1$ for the SM). In (32) the constants c_i are group theoretical factors that depend upon the embedding of the SM factors into a covering group, and warrant the same normalization for the covering group G and for the three group factors in the SM. For example, if the covering group is $SU(5)$, then $(c_Y, c_2, c_c) = (3/5, 1, 1)$, but they are different for other covering groups (see, for example, the table in [16]).

The three running coupling constants in α_i, may or may not converge into a single energy GUT scale M_G; if they do, then $\alpha_i(M_G) = \alpha$ is a constant independent of the index i. Now, for a embedding into a given covering group, c_i

are fixed values, and if we use for $F = 3$ (an experimental fact) and $H = 1$ as in the SM, then (32) constitutes a set of three equations with two unknowns, α and M_G, which may or may not have a consistent solution (more equations than unknowns).

The inputs to be used in (32) for $\alpha_i^{-1}(m_Z)$ are calculated from the experimental results [12]

$$\begin{aligned}\alpha_{em}^{-1}(m_Z) &= \alpha_Y^{-1}(m_Z) + \alpha_2^{-1}(m_Z)\\ &= 127.918 \pm 0.018\,,\\ \sin^2\theta_W(m_Z) &= 1 - \alpha_Y^{-1}(m_Z)\alpha_{em}(m_Z)\\ &= 0.23120 \pm 0.00015\,,\\ \alpha_c(m_Z) &= 0.1213 \pm 0.0018\,,\end{aligned}$$

which imply $\alpha_Y^{-1}(m_Z) = 98.343 \pm 0.036$, $\alpha_2^{-1}(m_Z) = 29.575 \pm 0.054$, and $\alpha_c^{-1}(m_Z) = 8.244 \pm 0.122$.

It is a well-known fact that the model based on the non-supersymmetric $SU(5)$ group of Georgi and Glashow [11] lacks gauge coupling unification because M_G is not unique in the range $10^{14}\,\mathrm{GeV} \leq M_G \leq 10^{16}\,\mathrm{GeV}$, predicting for the proton lifetime τ_p a value between 2.5×10^{28} years and 1.6×10^{30} years, which by the way is ruled out by experimental measurements [17]. If we introduce one more free parameter in the solutions to the RGE, as for example letting H become a free integer number, then we now have three unknowns with three equations that always have a mathematical solution (not necessarily with a physical meaning). Doing that in (32), we find that for $H = 7$ (seven Higgs doublets) we get the unique solution $M_G = 10^{13}\,\mathrm{GeV} \gg m_Z$ which, although a physical solution, is ruled out by the proton lifetime. So, if we still want unification, new physics at an intermediate mass scale M_V such that $m_Z < M_V < M_G$ must exist, supersymmetry (SUSY) being a popular candidate for that purpose [17].

The question now is if the 3-3-1 model under consideration in this paper introduces an intermediate mass scale M_V such that it achieves proper gauge coupling unification, being an alternative to SUSY. To answer this question using $SU(6)$ as the covering group as presented in Sect. 2, we must solve the following set of seven equations:

$$\begin{aligned}\alpha_i^{-1}(m_Z) &= \frac{\alpha_i^{-1}(M_V)}{c_i} - b_i(F,H)\ln\left(\frac{M_V}{m_Z}\right)\,,\\ \alpha_j^{-1}(M_V) &= \frac{\alpha^{-1}}{c'_j} - b'_j \ln\left(\frac{M_G}{M_V}\right)\,,\\ \alpha_Y^{-1}(M_V) &= \alpha_1^{-1}(M_V) + \alpha_3^{-1}(M_V)/3\,,\end{aligned} \tag{34}$$

where the last equation is just the matching conditions in (7), and $i = c, 2, Y$ and $j = c, 3, 1$ for the SM and the 3-3-1 model, respectively. The constants c_i are $(c_Y, c_2, c_3) = (3/5, 1, 1)$ as before, and $(c'_1, c'_3, c'_c) = (3/4, 1, 1)$, with the value $c'_1 = 3/4$ calculated from the electroweak mixing angle in (2). b'_j stand for the beta functions for the 3-3-1 model under study here.

Equation (34) constitutes a set of seven equations with seven unknowns $\alpha, \alpha_j(M_V), M_V, M_G$ and $\alpha_Y(M_V)$ [$\alpha_2(M_V) = \alpha_3(M_V)$ according to the matching conditions]. There is always a mathematical solution to this set of equations, but we want only physical solutions, that is solutions such that $m_Z < M_V < M_G$.

The new beta functions, calculated with the particle content introduced in Sect. 2, are

$$2\pi\begin{pmatrix} b'_1 \\ b'_3 \\ b'_c \end{pmatrix} = \begin{pmatrix} 0-8-7/9 \\ 11-4-4/6 \\ 11-6-0 \end{pmatrix} = \begin{pmatrix} -79/9 \\ 19/3 \\ 5 \end{pmatrix}\,, \tag{35}$$

where in the middle term we have separated the contributions coming from the gauge bosons, the fermion fields and the scalar fields, in that order. When we introduce these values in (34), we do not obtain a physical solution in the sense that we get $m_Z < M_G < M_V$.

Of course, if there are more particles at the 3-3-1 mass scale, then the beta functions given in (35) are not the full story. In particular, we know from Sect. 3 that at least new Higgs scalars are needed in order to generate a consistent lepton mass spectrum, so let us allow the presence in the model of the following Higgs scalar multiplets at the 3-3-1 mass scale: $N_X^{(1)}$ $SU(3)_L$ singlets (with $U(1)_X$ hypercharge equal to X), $N_X^{(3)}$ triplets (color singlets), $\tilde{N}_X^{(3)}$ leptoquark triplets (color triplets) and $N_X^{(6)}$ sextuplets (color singlets). These new particles will contribute to the beta functions b'_j in the following way:

$$2\pi\begin{pmatrix} b'_1 \\ b'_3 \\ b'_c \end{pmatrix} = \begin{pmatrix} -79/9 - \sum_X X^2 f(N_X^{(6)}, \tilde{N}_X^{(3)}, N_X^{(3)}, N_X^{(0)}) \\ 19/3 - \frac{1}{6}\sum_X (N_X^{(3)} + 3\tilde{N}_X^{(3)} + 5N_X^{(6)}) \\ 5 - \sum_X \tilde{N}_X^{(3)}/2 \end{pmatrix}\,, \tag{36}$$

where $f(N_X^{(6)}, \tilde{N}_X^{(3)}, N_X^{(3)}, N_X^{(0)}) = (2N_X^{(6)} + 3\tilde{N}_X^{(3)} + N_X^{(3)} + N_X^{(0)}/3)$; with these new $SU(3)_L$ multiplets contributing or not to the beta functions b_i of the SM factor groups, in agreement with the extended survival hypothesis [14] (for example, a sextuplet with a VEV $\langle \chi_{23}(1, 6, 2/3)\rangle \sim v$ contributes as an $SU(2)_L$ doublet in b_Y and b_2, etc.).

The calculation shows that for the following set of extra scalar Higgs fields that do not develop VEV: $N_X^{(0)} = 0$, $N_{1/3}^{(3)} = 1$, $N_{-2/3}^{(3)} = 1$, $\tilde{N}_X^{(3)} = 0$ $N_0^{(3)} = 21$ and $N_0^{(6)} = 9$, the set of equations in (36) has the physical solution

$$M_V \approx 1.9\,\mathrm{TeV} < M_G \approx 5 \times 10^8\,\mathrm{GeV}\,, \tag{37}$$

which provides a convenient 3-3-1 mass scale and a low unification GUT mass scale, as is shown in Fig. 3.

However, is this low GUT scale in conflict with the bounds on proton decay? The answer is no, because due to the Z_2 symmetry, our unifying group is $SU(6) \times Z_2$. Then we must assign to each irrep of $SU(6)$ in (9) a given Z_2 charge in accordance with the Z_2 value assigned to the 3-3-1 states in (18). For example, if we assign to one of the eight

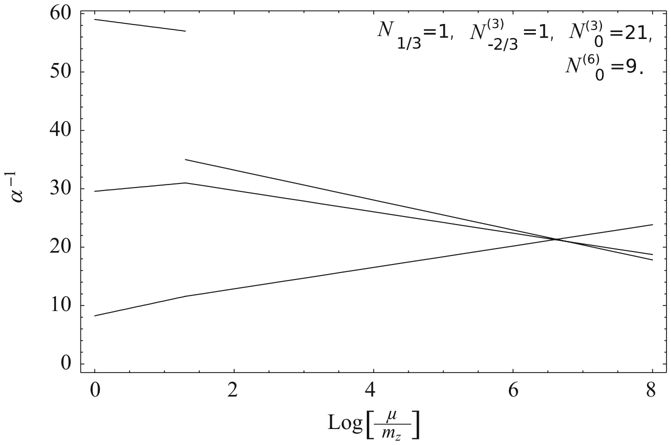

Fig. 3. Solutions to the RGE for the 3-3-1 model. For the meaning of $N_X^{(r)}$ see the main text

$\{6^*\} = \{D^c, -N_E^0, E^-, N_E^{0c}\}_L$ states in (9) a Z_2 value equal to 1, then we can perfectly identify D_L^c with one of the ordinary down quarks $(d^c, s^c, b^c)_L$, but then $(-N_E^0, E^-, N_E^{0c})_L$ cannot correspond to $(-\nu_l^0, l^-, \nu_l^{0c})_L$ because all of them have a Z_2 charge equal to zero; and the same for the other way around. As a consequence, the down quark d_L^c can not live together with $(\nu_e, e^-)_L$ in the same $SU(6) \times Z_2$ irrep, and the proton can not decay into light states belonging to the weak basis. The decay can of course occur via the mixing of ordinary 3-3-1 states with the extra new states in $SU(6)$, but such a mixing is of the order of $(M_V/M_G)^2$, which is a very small value. Of course, this argument is valid as far as we can find a mechanism able to produce GUT scale masses for all the extra states, but such an analysis is outside the scope of the present work.

5 Constraints on the parameters

In this section we shall set bounds on the mass of the new neutral gauge boson Z_2^μ, and its mixing angle θ with the ordinary neutral gauge boson, using the partial decay width for Z_1^μ. We shall also set constraints coming from possible FCNC effects and analyze the violation of unitarity of the Cabbibo–Kobayashi–Maskawa mixing matrix V^0_{CKM}.

Table 3. Experimental data and SM values for some parameters related to neutral currents

	Experimental results	SM
Γ_Z GeV	2.4952 ± 0.0023	2.4966 ± 0.0016
$\Gamma(had)$GeV	1.7444 ± 0.0020	1.7429 ± 0.0015
$\Gamma(l^+l^-)$GeV	83.984 ± 0.086	84.019 ± 0.027
R_e	20.804 ± 0.050	20.744 ± 0.018
R_μ	20.785 ± 0.033	20.744 ± 0.018
R_τ	20.764 ± 0.045	20.790 ± 0.018
R_b	0.21664 ± 0.00068	0.21569 ± 0.00016
R_c	0.1729 ± 0.0032	0.17230 ± 0.00007
Q_W^{Cs}	$-72.74 \pm 0.29 \pm 0.36$	-73.19 ± 0.13
M_{Z_1} GeV	91.1872 ± 0.0021	91.1870 ± 0.0021

5.1 Bounds on M_{Z_2} and θ

Let us notice to start with that, after the identification of the mass eigenstates, we can properly bound $\sin\theta$ and M_{Z_2} by using parameters measured at the Z pole from CERN e^+e^- collider (LEP), SLAC linear collider (SLC), and the atomic parity violation constraints given in Table 3.

The expression for the partial decay width for $Z_1^\mu \to f\bar{f}$ is

$$\begin{aligned}\Gamma(Z_1^\mu \to f\bar{f}) = &\frac{N_C G_F M_{Z_1}^3}{6\pi\sqrt{2}}\rho \\ &\times \left\{ \frac{3\beta - \beta^3}{2}[g(f)_{1V}]^2 + \beta^3 [g(f)_{1A}]^2 \right\} \\ &\times (1+\delta_f) R_{\mathrm{EW}} R_{\mathrm{QCD}} \ , \end{aligned} \quad (38)$$

where f is an ordinary SM fermion, Z_1^μ is the physical gauge boson observed at LEP, $N_C = 1$ for leptons while for quarks $N_C = 3(1 + \alpha_s/\pi + 1.405\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3)$, where the 3 is due to color and the factor in parenthesis represents the universal part of the QCD corrections for massless quarks (for fermion mass effects and further QCD corrections which are different for vector and axial-vector partial widths, see [18]); R_{EW} are the electroweak corrections including the leading order QED corrections given by $R_{\mathrm{QED}} = 1 + 3\alpha/(4\pi)$. R_{QCD} are further QCD corrections (for a comprehensive review see [19] and references therein), and $\beta = \sqrt{1 - 4m_f^2/M_{Z_1}^2}$ is a kinematic factor that can be taken equal to 1 for all the SM fermions except for the bottom quark. The factor δ_f contains the one loop vertex contribution which is negligible for all fermion fields except for the bottom quark, for which the contribution coming from the top quark, at the one loop vertex radiative correction, is parameterized as $\delta_b \approx 10^{-2}[-m_t^2/(2M_{Z_1}^2) + 1/5]$ [20]. The ρ parameter can be expanded as $\rho = 1 + \delta\rho_0 + \delta\rho_V$ where the oblique correction $\delta\rho_0$ is given by $\delta\rho_0 \approx 3G_F m_t^2/(8\pi^2\sqrt{2})$, and $\delta\rho_V$ is the tree-level contribution due to the $(Z_\mu - Z'_\mu)$ mixing which can be parameterized as $\delta\rho_V \approx (M_{Z_2}^2/M_{Z_1}^2 - 1)\sin^2\theta$. Finally, $g(f)_{1V}$ and $g(f)_{1A}$ are the coupling constants of the physical Z_1^μ field with ordinary fermions which, for this model, are listed in Table 1.

In the following we shall use the experimental values [12]: $M_{Z_1} = 91.188$ GeV, $m_t = 174.3$ GeV, $\alpha_s(m_Z) = 0.1192$, $\alpha(m_Z)^{-1} = 127.938$, and $\sin\theta_W^2 = 0.2333$. These values are introduced using the definitions $R_\eta \equiv \Gamma_Z(\eta\eta)/\Gamma_Z(\mathrm{hadrons})$ for $\eta = e, \mu, \tau, b, c, s, u, d$.

As a first result, notice from Table 1 that this model predicts $R_e = R_\mu = R_\tau$, in agreement with the experimental results in Table 3, independent of any flavor mixing at the tree-level.

The effective weak charge in atomic parity violation, Q_W, can be expressed as a function of the number of protons (Z) and the number of neutrons (N) in the atomic nucleus in the form

$$Q_W = -2\left[(2Z+N)c_{1u} + (Z+2N)c_{1d}\right], \quad (39)$$

where $c_{1q} = 2g(e)_{1A}g(q)_{1V}$. The theoretical value for Q_W for the cesium atom is given by [21] $Q_W(^{133}_{55}Cs) = -73.19 \pm 0.13 + \Delta Q_W$, where the contribution of new physics is included in ΔQ_W, which can be written as [22]

$$\Delta Q_W = \left[\left(1 + 4\frac{S_W^4}{1-2S_W^2}\right) Z - N\right]\delta\rho_V + \Delta Q'_W \ . \quad (40)$$

The term $\Delta Q'_W$ is model dependent and it can be obtained for our model by using $g(e)_{iA}$ and $g(q)_{iV}$, $i = 1, 2$, from Tables 1 and 2. The value that we obtain is

$$\Delta Q'_W = (3.75Z + 2.56N)\sin\theta + (1.22Z + 0.41N)\frac{M_{Z_1}^2}{M_{Z_2}^2} \ . \quad (41)$$

The discrepancy between SM and experimental data for ΔQ_W is given by [21]

$$\Delta Q_W = Q_W^{\rm exp} - Q_W^{\rm SM} = 0.45 \pm 0.48, \quad (42)$$

which is 1.1 σ away from the SM predictions.

Introducing the expressions for the Z pole observable in (38), with ΔQ_W in terms of new physics in (40) and using experimental data from LEP, SLC and atomic parity violation (see Table 3), we do a χ^2 fit and we find the best allowed region in the $(\theta - M_{Z_2})$ plane at 95% confidence level (CL). In Fig. 4 we display this region, which gives us the constraints

$$-0.00156 \le \theta \le 0.00105\,, \qquad 2.1 \text{ TeV} \le M_{Z_2} \ . \quad (43)$$

As we can see, the mass of the new neutral gauge boson is compatible with the bound obtained in $p\bar{p}$ collisions at the Fermilab Tevatron [23]. From our analysis we can also see that for $|\theta| \to 0$, M_{Z_2} peaks at a finite value larger than 100 TeV, which still copes with the experimental constraints on the ρ parameter.

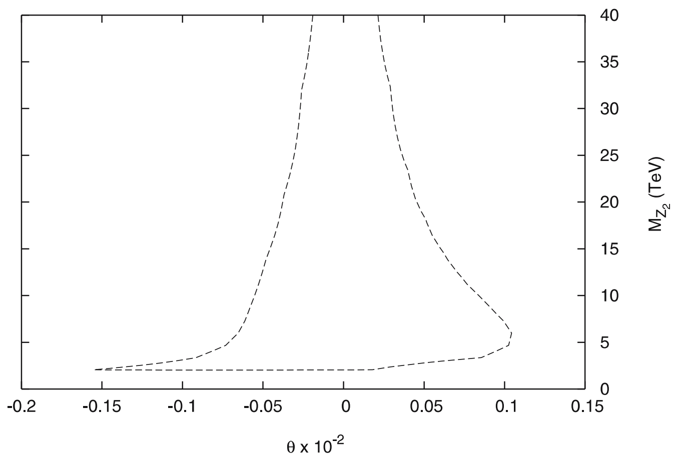

Fig. 4. Contour plot displaying the allowed region for θ vs M_{Z_2} at 95% CL

5.2 Bounds from unitarity violation of the CKM mixing matrix

The see-saw mass mixing matrices for quarks presented in (20) and (25) are not a consequence of the particular discrete Z_2 symmetry introduced in (18), in the sense that it is a straightforward calculation to show that any Z_n symmetry will reproduce the same quark mass matrices as long as we impose the following constraints:

- A pure see-saw mass matrix in the down quark sector.
- A tree-level mass entry for the top quark in the third family, plus a see-saw matrix for the other two families in the up quark sector.
- The non-minimal set of four Higgs scalar fields introduced in (6).

As a consequence of the mixing in the quark mass matrices, violation of unitarity of the CKM mixing matrix appears. Notice that for this particular model, $V^0_{\rm CKM}$ is obtained as the upper left 3×3 submatrix of a 4×5 mixing matrix (obtained, in turn, as the product of a 4×3 submatrix taken from the unitary 4×4 diagonalization matrix in the up quark sector with the fourth column suppressed, times a 3×5 submatrix of the unitary 5×5 diagonalization matrix of the down quark sector with the last two rows suppressed; all this as a consequence of having only three active quarks in the charged weak current $J^\mu_{W^+}$).

The unitarity violation arising in the model must be compatible with the experimental constraints on the CKM mixing parameters, as discussed, for example, in Section 11 of [12], where uncertainties in the third decimal place of the entries $V^0_{u_id_j}$ (the i,j element of $V^0_{\rm CKM}$), can be taken as possible signals of violation of unitarity.

Now, for the model discussed here, the structure of the quark mass matrices implies a mixing proportional to $\cos\delta$ (with $\delta = v/V$, as before) for the known quarks of each sector, which, when combined in the $V^0_{u_id_j}$ entries, gives a mixing of the form $\cos^2\delta = 1 - \sin^2\delta \approx 1 - \delta^2$, δ^2 being proportional to the violation of unitarity in the model. Taking for $V \approx M_{Z_2} \approx 2.1$ TeV (the lower bound in (43), we obtain $\delta^2 \approx 3.4\times10^{-3}$, which is in the limit of the allowed unitarity violation of $V^0_{\rm CKM}$ [12].

However, the former is not the full story, because violation of unitarity of $V^0_{\rm CKM}$ automatically induces FCNC. Unfortunately, violation of unitarity of $V^0_{\rm CKM}$ is not the best place to look for FCNC processes, because almost all the phenomenology of the CKM mixing matrix is done under the assumption of unitarity, which is not the case in the model presented here.

5.3 FCNC processes

In a model like this, with four scalar triplets and mixing of ordinary with exotic fermion fields, we should worry about possible FCNC effects.

First, notice that due to our Z_2 symmetry, FCNC effects do not occur at tree-level in the Lagrangian, because each flavor couples only to a single multiplet. However, FCNC effects can occur in $J_{\mu,L}(Z)$ and $J_{\mu,L}(Z')$ in (14) and (15), respectively, due again to the mixing of ordinary and heavy exotic fermion fields (notice from (14) that $J_{\mu,L}(Z)$ only includes the three ordinary up-type and down-type quarks as active quarks).

The best place to study the suppression of $d \leftrightarrow s$ currents is in the $(K_L^0 - K_S^0)$ mass difference, which may get contributions from the exchange of Z_1 and Z_2 between $d \leftrightarrow s$ currents. The contribution from Z_1 is proportional to $|V_{us}^{0\dagger} V_{ud}^0|^2 \approx |V_{us}^{0\dagger} V_{ud}^0|^2_{\rm SM} + 4\delta^4$ (where $|V_{us}^{0\dagger} V_{ud}^0|^2_{\rm SM}$ refers to the SM contribution, which is in agreement with the experimental data). Then, the mixing of light and heavy quarks implies extra FCNC effects proportional to $4\delta^4$, which, for $V \approx 2.1$ TeV as before, implies a contribution to new FCNC effects proportional to 1.2×10^{-5}. This value should be compared with the experimental bound $m(K_L) - m(K_S) \approx 3.48 \pm 0.006 \times 10^{-12}$ MeV [12]. Then, for $V \approx 2.1$ TeV we have $4\delta^4 \leq 0.006/3.48$, which means that there is room in the experimental uncertainties to include the new FCNC effects coming from violation of unitarity of the CKM mixing matrix present in the model.

Now, the contributions coming from Z_2 alone are safe, because they are not only constrained by the δ parameter, but also by the mixing angle $-0.00156 \leq \theta \leq 0.00105$ as given by (43).

6 Conclusions

During the last decade several 3-3-1 models for one and three families have been analyzed in the literature, the most popular one being the Pleitez–Frampton model [2], which is certainly not the only possible construction based on this local gauge group. Another two different three-family models, more appealing but not so popular in the literature, are introduced in [3] and [8, 24]. The model in [3], studied in this paper, contains right-handed neutrinos, while the model in [24] does not include right-handed neutrinos but has one extra exotic electron per family. Further, the analysis presented in [8, 25] shows that there is indeed an infinite number of anomaly-free models based on the 3-3-1 gauge structure, most of them including particles with exotic electric charges; but the number of models with particles without exotic electric charges are just a few. For example, another two 3-3-1 models for one family and only with particles of ordinary electric charge are analyzed in [26].

In this paper, we have carried out a systematic study of the so called 3-3-1 model with right handed neutrinos. Concretely, we have recalculated its charged and neutral currents, embedded the structure into $SU(6)$ as a covering group, looked for unification possibilities, studied the quark mass spectrum, and finally, by using updated precision measurements of the electroweak sector, we have set new limits for the mixing angle between the two heavy electrically neutral gauge weak bosons.

In our analysis, we have done a detailed study of the conditions that produce a consistent quark mass spectrum in the context of this model, an analysis only sketched in previous works [3], except for the neutral lepton sector [6]. First we have shown that a set of four Higgs scalars is enough to properly break the symmetry producing a consistent mass spectrum in the gauge boson sector. Then, the introduction of an appropriate anomaly-free discrete Z_2 symmetry allows us to construct an appealing mass spectrum in the quark sector without hierarchies in the Yukawa couplings. In particular we have carried a program in which: the three exotic quarks get heavy masses at the TeV scale; the top quark gets a tree-level mass at the electroweak scale; then the bottom, charm and strange quarks get see-saw masses, and finally, the first family quarks get radiative masses in such a way that $m_d \approx 2m_u$; the former without introducing strong hierarchies in the Yukawa coupling constants, or new mass scales in the model.

In addition, we have also embedded the model into the covering group $SU(6) \supset SU(5)$ and studied the conditions for gauge coupling unification at a scale $M_G \approx 5 \times 10^8$ GeV. The analysis has shown that a physical $(m_Z < M_V < M_G)$ one-loop solution to the RGE can be achieved at the expense of introducing extra Higgs scalars at the intermediate energy scale M_V.

The fact that the RGE produces a 3-3-1 mass scale of the same order (~ 2 TeV) as the lower limit obtained in the phenomenological analysis presented in Sect. 5 [compare (37) and (43)] is neither accidental nor fortuitous. As a matter of fact, the extra scalar fields contributing to the beta functions in (36), were introduced just for this job. A different set of scalar Higgs fields will produce either different 3-3-1 and GUT mass scales, no unification at all, or unphysical solutions. Even though our analysis may look a little arbitrary, we emphasize that we make the decision to play only with the most obscure part of any local gauge theory: the Higgs scalar sector.

Finally, we want to stress that, as discussed in the previous section, the lower bound 2.1 TeV for the mass of the new neutral gauge boson Z_2^μ is compatible with the constraints coming from violation of unitarity of the CKM mixing matrix and from new contributions to FCNC processes.

Acknowledgements. We acknowledge partial financial support from DIME at Universidad Nacional de Colombia-Sede Medellín, and from CODI at Universidad de Antioquia. W.A. Ponce thanks the Laboratorio de física teórica de la Universidad de La Plata in La Plata, Argentina, where part of this work was done. We also thank C. García-Canal, Huner Fanchiotti and Enrico Nardi for illuminating discussions.

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